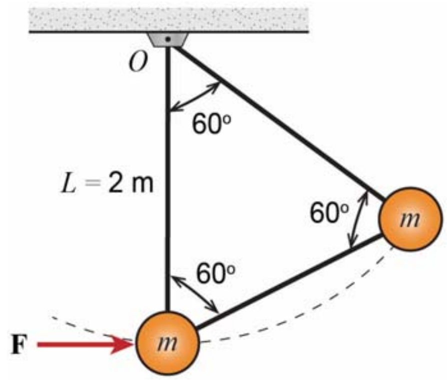
Solved Problem 9.7-6

Consider the symmetric triangular pendulum shown. The pendulum consists of two 5-kg masses connected by beams of length L = 2 meters and negligible mass. The pendulum starts from rest in the position shown where the left beam is completely vertical when a constant 1000-N force F is applied to the left bob of the pendulum for a period of 0.03 seconds. Determine the angular velocity of the pendulum immediately following the application of F and the maximum angle through which the pendulum swings before coming momentarily to rest.



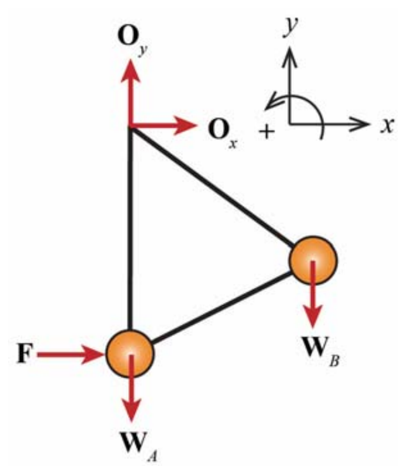
Given: m = 5 kg L = 2 m
$\omega_1 = 0$ @ $t = 0$ F = 1000 N
Δt = 0.03 seconds

Find: angular velocity after impact, ω_2 (at state 2)
maximum angle pendulum swings through, θ (at state 3)

Solution:

Getting familiar with the system

Since the force is applied over a known duration of time, we will employ an impulse-momentum approach to determine the angular velocity of the pendulum following the application of the impulse.



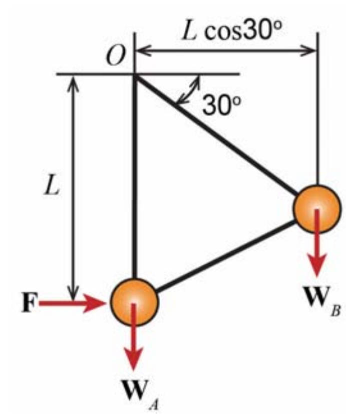
Free-body diagram

The first thing that we need to do is to draw a free-body diagram in order to determine which forces/moments apply external impulse to the system. Since the external force F is applied for such a short period of time, we will assume that the pendulum's position doesn't change during its application. We will apply the angular-impulse momentum principle with respect to point O. The reaction forces at O and the weight $\mathbf{W}_A$ don't impart a moment about point O.

$$\int_{t_1}^{t_2} \mathbf{M}_O dt = \mathbf{H}_{O,2} - \mathbf{H}_{O,1}$$

Angular impulse

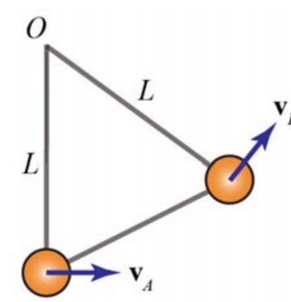
In calculating the angular impulse, force $\mathbf{F}$ and weight $\mathbf{W}_B$ are the two forces that impart a moment about the reference point O. These forces are constant, and based on the assumption that the pendulum remains approximately in the same location during the application of $\mathbf{F}$, we will treat the moment arms as constant also.



$$\int_{t_1}^{t_2} \mathbf{M}_O dt = \int_{t_1}^{t_2} (FL - W_B L\cos(30))\,\mathbf{k}\,dt = (FL - W_B L\cos(30))\,\Delta t\,\mathbf{k} \quad (1)$$

Angular momentum

Since we have a system of particles, the angular momentum of the system is the sum of the angular momenta of the individual particles (the two bobs). Also, the angular momentum of the system is initially zero since the system begins from rest.



$$\mathbf{H}_{O,2} - \mathbf{H}_{O,1} = \sum \mathbf{r}_{i,2} \times m_i \mathbf{v}_{i,2} - \sum \mathbf{r}_{i,1} \times m_i \cancel{\mathbf{v}_{i,1}} = \mathbf{r}_{A,2} \times m_A \mathbf{v}_{A,2} + \mathbf{r}_{B,2} \times m_B \mathbf{v}_{B,2}$$

Recognizing that the pendulum rotates about a fixed axis through O, each bob will move in a circular path about O with a speed $v = r\omega$ in a direction perpendicular to the vector pointing from O toward the bob.

$$\mathbf{H}_{O,2} - \mathbf{H}_{O,1} = (L^2 m_A \omega_2 + L^2 m_B \omega_2)\,\mathbf{k} \quad (2)$$

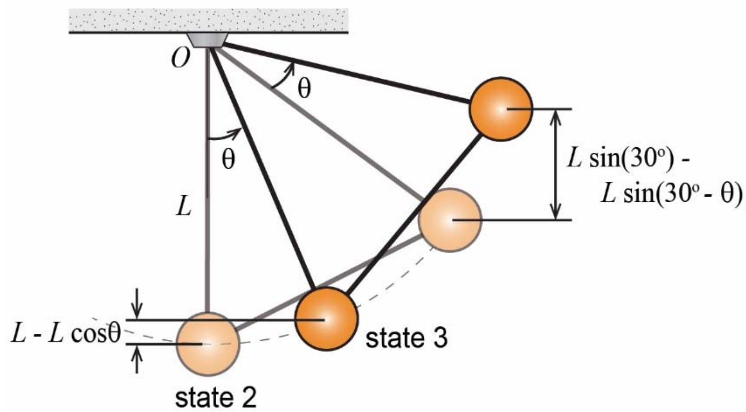
Angular impulse-momentum principle

Applying the angular impulse-momentum principle, we set Equation (1) and Equation (2) equal to one another and solve for the angular velocity ω of the system following the application of the external force F.

$$\int_{t_1}^{t_2} \mathbf{M}_O dt = \mathbf{H}_{O,2} - \mathbf{H}_{O,1} \qquad (FL - W_B L\cos(30))\Delta t = (L^2 m_A \omega_2 + L^2 m_B \omega_2)$$

$$\omega_2 = \frac{(F - W_B\cos(30))\Delta t}{L(m_A + m_B)} = \boxed{1.436\ \frac{\text{rad}}{\text{s}}\ \text{ccw}}$$

Conservation of energy



Once the external force F is released, the only forces acting on the pendulum are either conservative (the weights) or do no work (the reaction forces). Therefore, energy is conserved as the pendulum swings upward following the application of the impulsive force. Furthermore, the kinetic energy at state 3, when the pendulum reaches its maximum height, is zero since the system momentarily comes to rest. As with angular momentum, the energy of a system of particles (kinetic and potential) is simply the sum of the energies of the individual particles.

$$T_2 + \cancel{V_2} = \cancel{T_3} + V_3$$

$$\frac{1}{2}m_A v_{A,2}^2 + \frac{1}{2}m_B v_{B,2}^2 = m_A g\Delta h_{A,2-3} + m_B g \Delta h_{B,2-3} = m_A gL(1-\cos\theta) + m_B gL(\sin 30^o - \sin(30^o - \theta))$$

Employing the kinematic relationship, $v = r\omega$ and the value for angular velocity at state 2 solved for earlier, we can solve for the maximum angle θ. To solve the resulting nonlinear equation for θ you can use numerical techniques (e.g. a graphing calculator). Be careful to determine whether or not your software expects the angle to be in radians or not. The program I used needed the angle to be in radians so 30 degrees in the above equation was changed to $\pi/6$ radians before the solver was implemented.

$$\frac{1}{2}m_A L^2\omega_2^2 + \frac{1}{2}m_B L^2\omega_2^2 = m_A gL(1-\cos\theta) + m_B gL(\sin(\pi/6) - \sin((\pi/6)-\theta))$$

$$\cos\theta - \sin(\pi/6) + \sin((\pi/6)-\theta) + \frac{L\omega_2^2}{g} - 1 = 0 \qquad \boxed{\theta = 0.3742 \text{ rad} = 21.4^o}$$