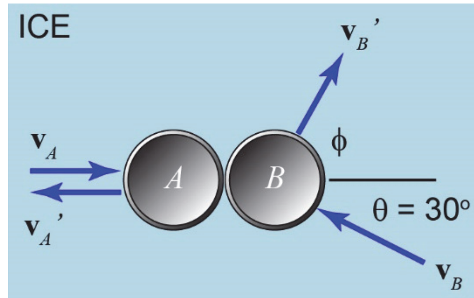
Solved Problem 9.4-5

Two identical hockey pucks moving with initial speeds of v_A = 6 m/s and v_B = 10 m/s, in the directions shown, collide. If the pucks move in the directions shown following the collision, and v_A' = 7 m/s, determine the coefficient of restitution and the angle ϕ.



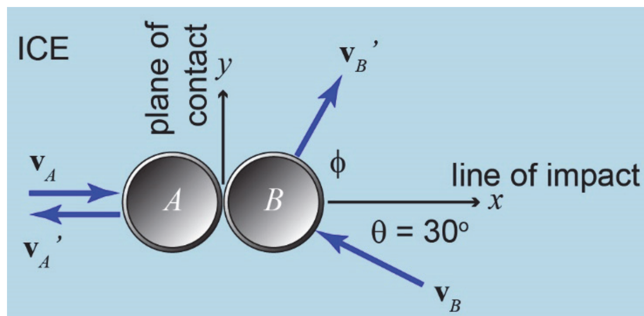
Given: $m_A = m_B = m$ $\quad$ v_A = 6 m/s
v_B = 10 m/s $\quad$ v_A' = 7 m/s
θ = 30°

Find: e, ϕ

Solution

Setting up the problem

When solving impact problems, it is helpful to apply the momentum equation in two special coordinate directions. These directions correspond to the *line of impact* and the *plane of contact*. Before we start our analysis, we should identify these two directions. The system conserves momentum in the direction of the line of impact, while each particle conserves momentum in the direction of the plane of contact.



Plane of contact

The momentum of the individual particles is conserved in the direction of the plane of contact.

$$mv_{Ay} = mv'_{Ay} \qquad v'_{Ay} = 0$$

$$mv_{By} = mv'_{By} \qquad v'_{By} = v_B \sin\theta = 5\ \frac{\text{m}}{\text{s}}$$

Line of impact

The momentum of the system is conserved in the direction of the line of impact. Remember that the velocity direction is important in the following equation. Use a plus or minus to indicate the direction along the coordinate axis.

$$mv_{Ax} + mv_{Bx} = mv'_{Ax} + mv'_{Bx} \qquad v_A - v_B\cos\theta = -v'_A + v'_{Bx}$$

$$6-10\cos 30=-7+v'_{Bx} \qquad v'_{Bx}=4.34\ \frac{\text{m}}{\text{s}}$$

Coefficient of restitution

Now that we know the x-direction velocity of puck B after the impact, we can calculate the coefficient of restitution.

$$e=\frac{v'_{Bx}-v'_{Ax}}{v_{Ax}-v_{Bx}}=\frac{4.34-7}{6-10\cos 30} \qquad \boxed{e\approx 1}$$

Angle

$$\phi=\tan^{-1}\frac{v'_{By}}{v'_{Bx}}=\tan^{-1}\frac{5}{4.34} \qquad \boxed{\phi=49^o}$$

9.5) ANGULAR MOMENTUM

Angular momentum is the rotational counterpart to *linear momentum* ($\mathbf{G} = m\mathbf{v}$). The units of angular momentum and linear momentum are different. Therefore, they are not compatible and cannot be added or subtracted. Angular momentum describes how an object moves around a specified point. All moving objects have angular momentum with respect to some point, but it is generally used to describe rotating bodies.

> Units of Angular Momentum
> SI units:
> - kilogram-meter^2 per second [kg-m^2/s]
> - Newton-meter-second [N-m-s]
>
> US customary units:
> - slug-foot^2 per second [slug-ft^2/s]
> - foot-pound-second [ft-lb-s]

The **angular momentum** $\mathbf{H}_O$ of a particle is defined as the moment of the linear momentum ($\mathbf{G} = m\mathbf{v}$) about point O. Similar to the moment produced by a force ($\mathbf{M}=\mathbf{r}\times\mathbf{F}$), the moment of the linear momentum is a function of the moment arm ($\mathbf{r}$) at which the linear momentum ($m\mathbf{v}$) is acting with respect to point O. Angular momentum is a different type of quantity than linear momentum even though they are related. This is analogous to how the moment induced by a force is a fundamentally different thing than the force itself. Particles moving about a point in a curved or straight path can have angular momentum with respect to a reference point. In the chapter on *Rigid-Body Impulse & Momentum,* we will also see that rigid bodies that are rotating or translating may have angular momentum. Whether or not a body has angular momentum depends on the choice of reference point. Mathematically, angular momentum is defined by the cross product relationship shown in Equation 9.5-1.

Angular momentum of a particle: $$\boxed{\mathbf{H}_O=\mathbf{r}\times\mathbf{G}=\mathbf{r}\times m\mathbf{v}} \quad (9.5\text{-}1)$$

$\mathbf{H}_o$ = angular momentum with respect to O
$\mathbf{r}$ = particle's position with respect to O
m = mass of the particle
$\mathbf{v}$ = velocity of the particle