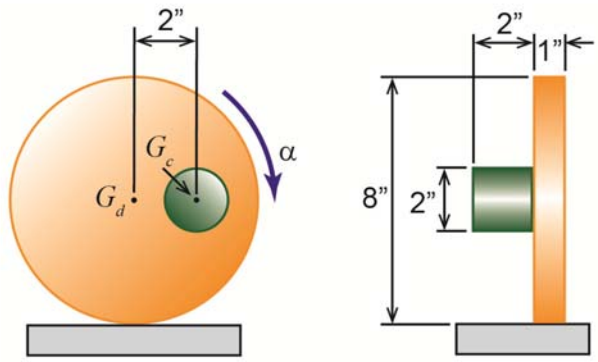
Example Problem 6.7-6

A 10-lb disk with a mass imbalance rolls on the ground. It is released from rest in the position shown. The mass imbalance is produced by an off-center 2-lb cylinder. Will the disk slip relative to the ground? The coefficient of static and kinetic friction between the disk and ground are 0.6 and 0.4, respectively.



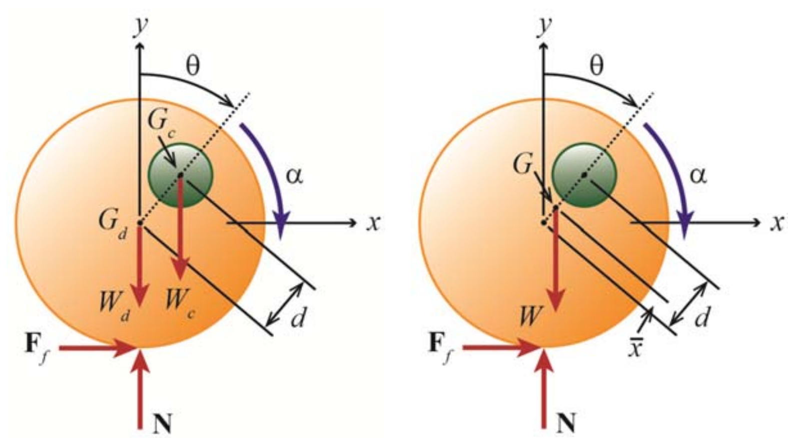
Given: $W_{cylinder}$ = 2 lb W_{disk} = 10 lb
W = 12 lb ω = 0
μ_s = 0.6 μ_k = 0.4
$r_{cylinder}$ = 1 in r_{disk} = 4 in
d = 2 in $\theta = 90^{o}$

Find: Will the disk slip?

Solution:

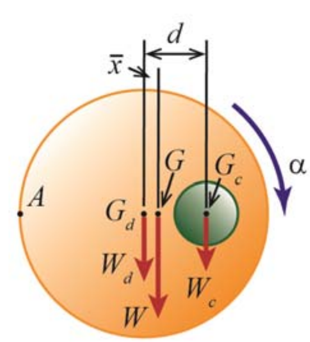
Free-body diagram

To start we will draw a free-body diagram making sure that we draw the cylinder at an angle. This will introduce the angular position variable (θ) that we need for the support reaction results. The two FBD's shown are equivalent. One uses the weights of the individual components and one uses the weight of the system. It turns out that it will be easier to analyze the motion of this system if we use the overall center of mass.



Center of mass

The easiest way to find the overall center of mass is to balance the moments. Consider the figure that shows the weights of the individual components and the overall weight. The moments created by the individual components has to equal the moment induced by the overall weight.



$$\sum M_A = W_{disk} r_{disk} + W_{cylinder}(r_{disk} + d) = W(r_{disk} + \bar{x})$$

$$\bar{x} = \frac{W_{disk} r_{disk} + W_{cylinder}(r_{disk} + d)}{W} - r_{disk} = 0.333 \text{ in}$$

Mass moment of inertia

$$I_{disk,Gd} = \frac{m_{disk} r_{disk}^2}{2} \qquad I_{cylinder,Gc} = \frac{m_{cylinder} r_{cylinder}^2}{2}$$

Note that the tables only give the mass moment of inertias for a cylinder and disk about their respective center axes. Therefore, we will have to use the parallel axis theorem to calculate the inertias with respect to an axis parallel to the axis passing through G.

$$I_{cylinder,G} = I_{cylinder,Gc} + m_{cylinder}(d - \bar{x})^2 = \frac{m_{cylinder} r_{cylinder}^2}{2} + m_{cylinder}(d - \bar{x})^2$$

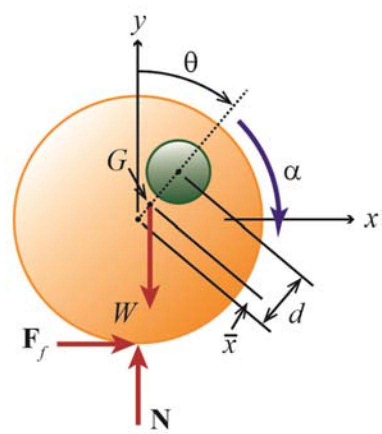
$$I_{disk,G} = I_{disk,Gc} + m_{disk}\bar{x}^2 = \frac{m_{disk} r_{disk}^2}{2} + m_{disk}\bar{x}^2$$

Now that both moments of inertia are about the same axis, they can be added.

$$I_G = I_{cylinder,G} + I_{disk,G} = \frac{m_{cylinder} r_{cylinder}^2}{2} + m_{cylinder}(d - \bar{x})^2 + \frac{m_{disk} r_{disk}^2}{2} + m_{disk}\bar{x}^2 = 2.723 \text{ slug}\cdot\text{in}^2$$

Equation of motion

We will use Newton's second law and a form of Euler's second law to determine the motion of the disk and the reaction forces. We will start with summing the moments. Since this is a case of general planar motion we will use the center of mass of the system as a reference.



$$\sum \mathbf{M}_G = I_G \boldsymbol{\alpha} \qquad -F_f r_{disk} + N\,\bar{x}\sin\theta = I_G \alpha$$

$$\alpha = \frac{-F_f r_{disk} + N\,\bar{x}\sin\theta}{I_G} = \frac{-F_f r_{disk} + N\,\bar{x}}{I_G} \qquad (1)$$

Kinematics

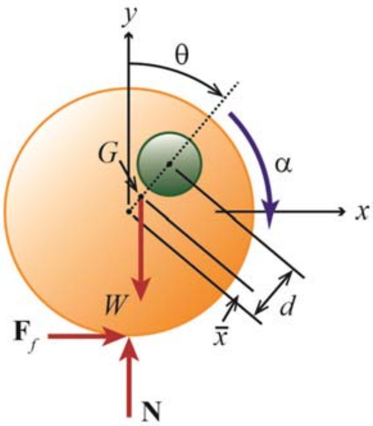
We don't, at this moment, know whether the friction is static or kinetic. Let's assume that there is no slip and then check our assumption at the end. If there is no slip, the acceleration of the center of the disk may be determined by the following equation.

$$a_{Gd,x} = r_{disk}\alpha$$

The acceleration of the overall center of gravity may be determined with the following equation. Remember that the system is released from rest (i.e ω = 0) and the disk is in the θ = 90^o position.

$$\begin{aligned}
\mathbf{a}_G &= \mathbf{a}_{Gd} + \boldsymbol{\alpha}\times\mathbf{r}_{G/Gd} - \omega^2\mathbf{r}_{G/Gd} \\
&= r_{disk}\alpha\,\mathbf{i} + \alpha(-\mathbf{k})\times\bar{x}(\sin\theta\,\mathbf{i}+\cos\theta\,\mathbf{j}) - \omega^2\bar{x}(\sin\theta\,\mathbf{i}+\cos\theta\,\mathbf{j}) \\
&= r_{disk}\alpha\,\mathbf{i} + \alpha\,\bar{x}(-\sin\theta\,\mathbf{j}+\cos\theta\,\mathbf{i}) - \phi^2\bar{x}(\sin\theta\,\mathbf{i}+\cos\theta\,\mathbf{j}) \\
&= \alpha(\bar{x}\cos\theta + r_{disk})\,\mathbf{i} - \alpha\,\bar{x}\sin\theta\,\mathbf{j} \\
&= \alpha\, r_{disk}\,\mathbf{i} - \alpha\,\bar{x}\,\mathbf{j}
\end{aligned}$$

(2)



Equation of motion

We can now apply Newton's second law to determine the friction force and normal force at the ground contact. We will use the angular acceleration given in Equation (1) and the acceleration given in Equation (2).

$$\sum\mathbf{F} = \sum m_i\mathbf{a}_{G,i}$$

<u>x-direction</u>

$$F_{fs} = \frac{W}{g}a_{Gx} = \frac{W}{g}\left[\alpha\, r_{disk}\right] = \frac{W r_{disk}}{g}\left[\frac{-F_{fs}r_{disk} + N\,\bar{x}}{I_G}\right] = 0.1823N - 2.1898F_{fs} \qquad (3)$$

<u>y-direction</u>

$$N - W = \frac{W}{g}a_{Gy} = \frac{W}{g}\left[-\alpha\,\bar{x}\right] = \frac{W\,\bar{x}}{g}\left[\frac{F_{fs}r_{disk} - N\,\bar{x}}{I_G}\right] \qquad N\left[1+\frac{W\,\bar{x}^2}{gI_G}\right] = W\left[1+\frac{F_{fs}r_{disk}\,\bar{x}}{gI_G}\right]$$

$$N = W\left[\frac{gI_G + F_{fs}r_{disk}\,\bar{x}}{gI_G + W\,\bar{x}^2}\right] = 11.82 + 0.1796F_{fs} \qquad (4)$$

Substituting Equation (4) into Equation (3) we get an expression for the friction force and use that to calculate the normal force.

$$F_{fs} = (0.1823(11.82+0.1796F_{fs}) - 2.1898F_{fs}) \qquad F_{fs} = 0.68 \text{ lb} \qquad N = 11.94 \text{ lb}$$

Maximum static friction force

$$F_{fs,\max} = \mu_s N = 7.17 \text{ lb} > F_{fs}$$

Since the maximum static friction force is greater than the static friction force calculated, **the disk will not slip relative to the ground**.