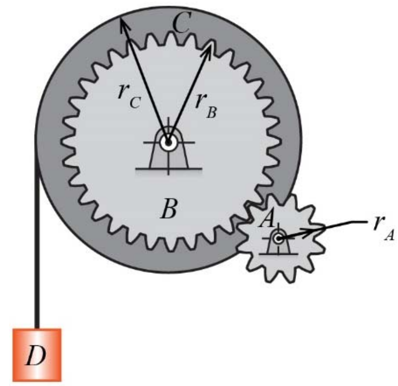
Solved Problem 6.6-5

It is desired that the shown hoisting mechanism be operated such that the load D (W = 10 lb) is lifted at a constant acceleration of 1 ft/s^2. If the drum C, which is rigidly attached to gear B, has a radius of r_C = 4 ft and the gear ratio between B and A is 3:1, determine the torque with which a motor must drive pinion gear A. (Note that the mass moments of inertia for the drum C/Gear B combination and gear A about their axes of rotation are I_B = 40 slug-ft^2, I_A = 10 slug-ft^2)



Given: W = 10 lb, a_D = 1 ft/s^2
r_C = 4 ft, r_B/r_A = 3
I_B = 40 slug-ft^2
I_A = 10 slug-ft^2

Find: M_A

Solution:

Getting familiar with the problem

Reading the problem statement, we know that gear A is the driving gear and gear B is the driven gear. This means that gear A will have an external moment from the motor applied to it and gear B will not. What drives gear B is the force that gear A's teeth apply to gear B's teeth. Both gears have a mass moment and, therefore, will influence the angular acceleration of the system. Furthermore, both gears rotate about a fixed axis. Therefore, we will use the fixed axis version of Euler's second law to determine the relationship between the moments and angular accelerations (i.e. $\sum \mathbf{M} = I_O \boldsymbol{\alpha}$).

Angular accelerations

The only acceleration information given in the problem statement is the linear acceleration of load D. We will need to use this to calculate the angular acceleration of drum C. Noting that the acceleration of load D is the tangential acceleration of any circumferential point on drum C.

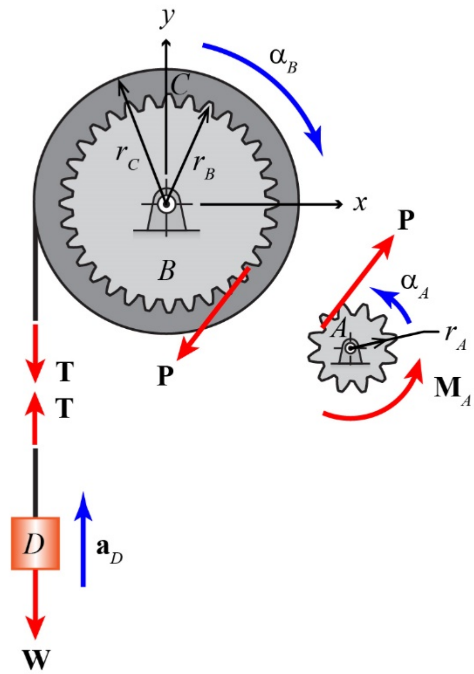
$$a_D = r_C \alpha_B \qquad \alpha_B = 0.25 \ \frac{\text{rad}}{\text{s}^2}$$

We can then use the gear ratio to find the angular acceleration of gear A.

$$\alpha_A = \frac{r_B}{r_A}\alpha_B = 0.75 \ \frac{\text{rad}}{\text{s}^2}$$

Free-body diagram

The next step will be to draw a free-body diagram of each component of the system.



Equations of motion

After completing the FBD, we will apply Newton's laws and Euler's second law to each component of the system to determine the motor torque.

Load D

$$\sum F_y = m_D a_{Dy} \qquad T - W = \frac{W}{g} a_D$$

$$T = W\left(\frac{a_D}{g} + 1\right) = 10.31 \text{ lb}$$

(Remember to use g = 32.2 ft/s^2)

Gear B

$$\sum \mathbf{M}_B = I_B \boldsymbol{\alpha}_B \qquad -Tr_C + Pr_B = I_B \alpha_B \qquad P = \frac{I_B \alpha_B}{r_B} + \frac{Tr_C}{r_B}$$

Gear A

$$\sum \mathbf{M}_A = I_A \boldsymbol{\alpha}_A \qquad M_A - Pr_A = I_A \alpha_A$$

Substituting P from the gear B equation we get

$$M_A - \left(\frac{I_B \alpha_B}{r_B} + \frac{Tr_C}{r_B}\right) r_A = I_A \alpha_A \qquad M_A = I_A \alpha_A + \left(I_B \alpha_B + Tr_C\right)\frac{r_A}{r_B}$$

$$\boxed{M_A = 24.6 \text{ ft-lb}}$$