Solved Problem 3.3-12

A car drives along a curvy mountain road. The driver uses the gas pedal to uniformly accelerate the car from 20 mph to 40 mph in 30 seconds around a constant 1000-ft radius curve. Determine the car's acceleration as a function of time.

Given: v_o = 20 mph = 29.33 ft/s, v_f = 40 mph = 58.67 ft/s
Δt = 30 s, ρ = 1000 ft

Find: $\mathbf{a}$

Solution:

Getting familiar with the problem

This problem mentions three things that should steer you in the direction of using the n-t coordinate system.

1. It mentions a curvy road. The x-y coordinate system is not ideal for use on curvy motion, especially when the x- and y-directions are coupled.
2. It mentions and acceleration in the direction of motion (i.e. the driver uses the gas pedal to accelerate) or a tangential acceleration.
3. It also mentions the radius of the curve that the car is traveling on.

The first thing you should do once you have decided to use the n-t coordinate system is to write the acceleration equation down. This gives you a sense of what quantities you have, which you need to find, and most of all, it reminds you that there are two components to the acceleration.

$$\mathbf{a} = \dot{v}\,\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n$$

Tangential acceleration

The tangential acceleration may be calculated by applying the average acceleration equation in the tangential direction.

$$a_t = \dot{v} = \frac{\Delta v}{\Delta t} = \frac{58.67 - 29.33}{30} = 0.98\ \frac{\text{ft}}{\text{s}^2}$$

Velocity

The velocity is always in the tangential direction, therefore, we may use the constant acceleration equation (because a_t = constant) to determine the velocity as a function of time.

$$v = a(t - t_o) + v_o \qquad v = a_t t + v_o = 0.98t + 29.33\ \frac{\text{ft}}{\text{s}}$$

Total acceleration

Using the normal and tangential acceleration equation, we can determine the total acceleration of the car.

$$\mathbf{a} = \dot{v}\,\mathbf{e}_t + \frac{v^2}{\rho}\mathbf{e}_n \qquad \boxed{\mathbf{a} = 0.98\,\mathbf{e}_t + \frac{(0.98t + 29.33)^2}{1000}\mathbf{e}_n}$$

To get a sense of how much the normal acceleration increases as the velocity increases, let's construct a plot of the tangential and normal accelerations of the car.