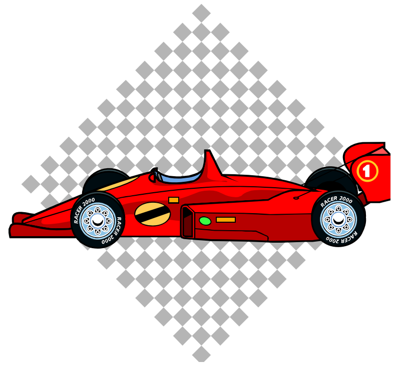
Solved Problem 2.4-5

A drag race between two cars takes place on a flat straight race track. The two competing cars have maximum acceleration and velocity capabilities of $a_{max,1}$ = 36 ft/s^2, $v_{max,1}$ = 150 mph and $a_{max,2}$ = 38 ft/s^2, $v_{max,2}$ = 140 mph, respectively. Consider that each car starts from rest, accelerates at its maximum rate until it reaches its maximum speed. After reaching their maximum speed, the cars continues with constant velocity until they reach the finish line. If the track is s_f = 1/4 mi long, which car will win the race and by how much time?



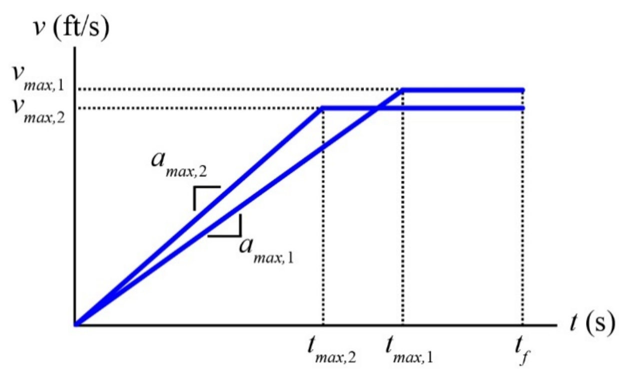
Given: $v_{max,1}$ = 150 mph = 220 ft/s
$v_{max,2}$ = 140 mph = 205.33 ft/s
$a_{max,1}$ = 36 ft/s^2
$a_{max,2}$ = 38 ft/s^2
s_f = 1/4 mi = 1320 ft

Find: Which car will win the race? $t_{f,1} - t_{f,2}$

Solution:

Setting up the problem

To get a visual sense of the problem, let's start by graphing the velocity of the two cars using variables for any unknown quantities. We know that the cars will accelerate up to their maximum velocity and then continue at that maximum velocity until the end of the race. The slope of the line that goes from the start of the race (v = 0) to the maximum velocity (v_{max}) is equal to the acceleration.



Time to get to the maximum velocity

The time to get up to the car's maximum velocity is determined by the slope (acceleration) of the initial portion of the v-t graph.

$$a_{\max} = \frac{v_{\max}}{t_{\max}} \qquad t_{\max} = \frac{v_{\max}}{a_{\max}}$$

$t_{max,1}$ = 6.11 s $\qquad$ $t_{max,2}$ = 5.40 s

Time to finish the race

To determine the time that it takes each car to finish the race, use the fact that the area under the v-t curve is equal to the displacement.

$$s_f = \frac{1}{2} t_{\max} v_{\max} + v_{\max}(t_f - t_{\max}) \qquad t_f = \frac{s_f}{v_{\max}} + \frac{1}{2} t_{\max}$$

$t_{f,1}$ = 9.06 s $\qquad$ $t_{f,2}$ = 9.13 s

Car 1 will win by **0.07 s**.